Lecture-2

Ampere's circuit law, Maxwell's equation, application of ampere's law, magnetic flux density- Maxwell's equation, Maxwell's equation for static fields, magnetic scalar and vector potential.

Pradeep Singla

What is Electromagnetics?

What is the basis of electromagnetics ? CHARGE

Electromagnetics is the study of CHARGES

at rest / in motion

The subject electromagnetics may be divided into 3 branches:

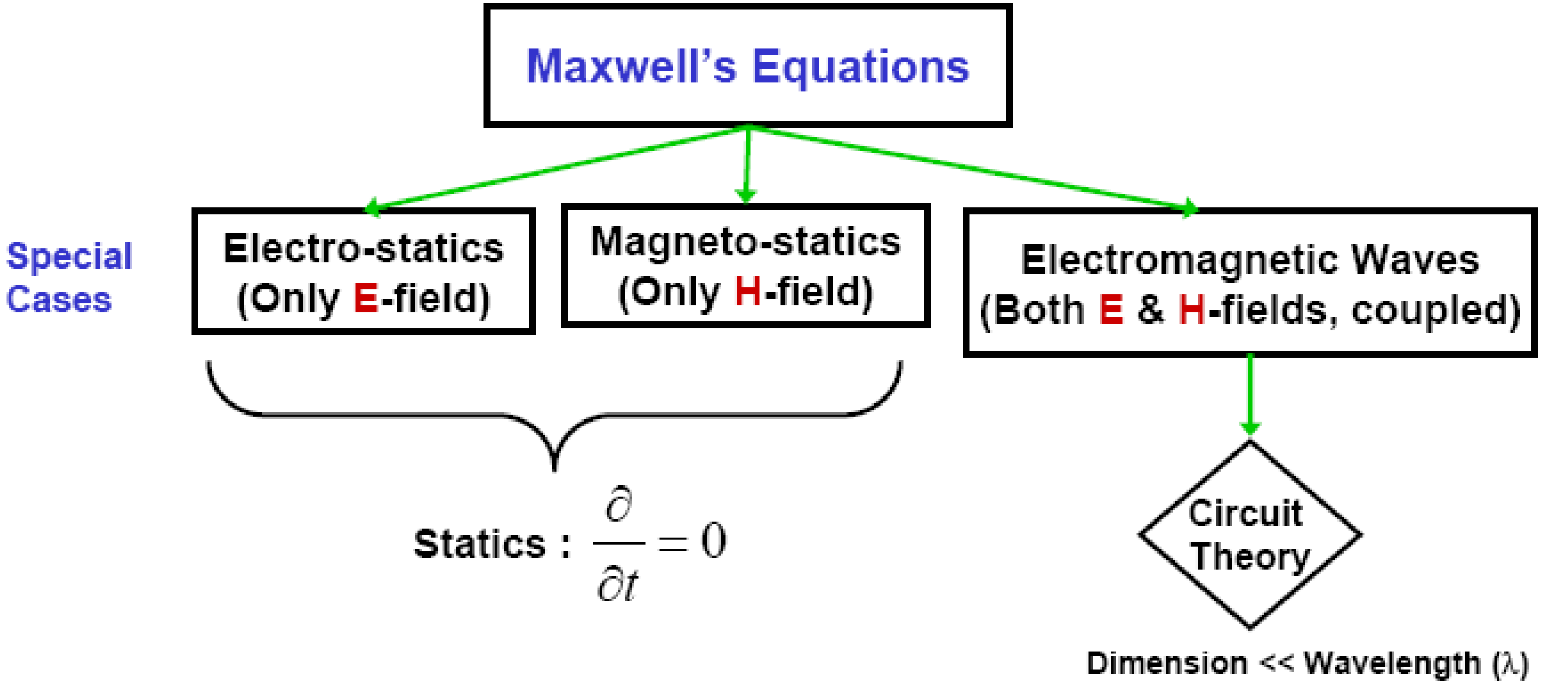
Electrostatics : charges are at rest (no time-variation)

Magnetostatics : charges are in steady-motion (no time-variation)

Electrodynamics : charges are in time-varying motion

(give rise to waves that propagate and carry energy and information)

Fundamental Laws of Electromagnetics



E-Static *vs* M-Static

Attribute	Electrostatics	Magnetostatics
Sources	Stationary charges	Steady currents
Constitutive parameter (s)	ε and σ	μ
Equations *Differential form* *Integral form*	$\nabla \cdot \mathbf{D} = \rho_v$; $\nabla \times \mathbf{E} = 0$ $\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$; $\oint_C \mathbf{E} \cdot d\mathbf{l} = 0$	$\nabla \cdot \mathbf{B} = 0$; $\nabla \times \mathbf{H} = J$ $\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$; $\oint_C \mathbf{H} \cdot d\mathbf{l} = I$
Potential	$\mathbf{E} = -\nabla V$	$\mathbf{B} = \nabla \times \mathbf{A}$
Energy density	$w_e = ½\, \varepsilon E^2$	$w_m = ½\, \mu H^2$
Force on charge q	$\mathbf{F}_e = q\mathbf{E}$	$\mathbf{F}_m = q\mathbf{u} \times \mathbf{B}$
Circuit element (s)	C and R	L

The Story of E and B

- Stationary charges cause electric fields (Coulombs Law, Gauss' Law).

- Moving charges or currents cause magnetic fields (Biot-Savart Law). Therefore, electric fields produce magnetic fields.

- Question: Can changing magnetic fields cause electric fields?

Maxwell equations :

$$\mathrm{rot}\,\boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$ → **Time-independent case : (rt) law**

$$\mathrm{rot}\,\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$ → **(F) law of induction**

(T s)

$$\mathrm{div}\,\boldsymbol{D} = \rho$$ → **( ) law**

$$\mathrm{div}\,\boldsymbol{B} = 0$$ → **( ) law for magnetism**

( )

$$\boldsymbol{D} = \varepsilon \boldsymbol{E}$$
$$\boldsymbol{B} = \mu \boldsymbol{H}$$
$$\boldsymbol{J} = \sigma \boldsymbol{E}$$

( )

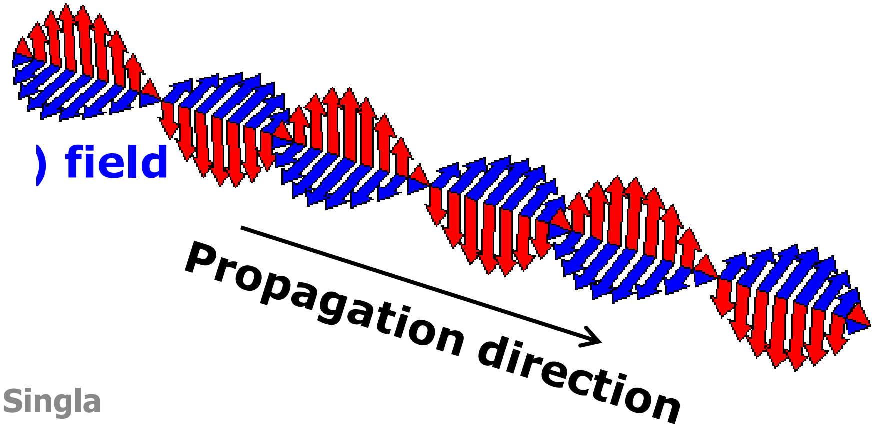
Electromagnetic wave :

propagation speed :

in a vacuum, $v =$

( ) field

(M) field



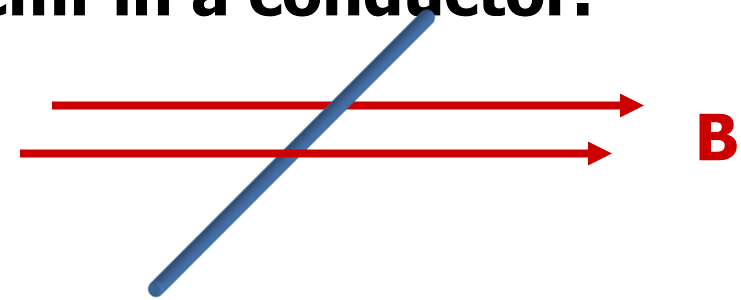
Induced emf and Faraday's Law

Magnetic Induction

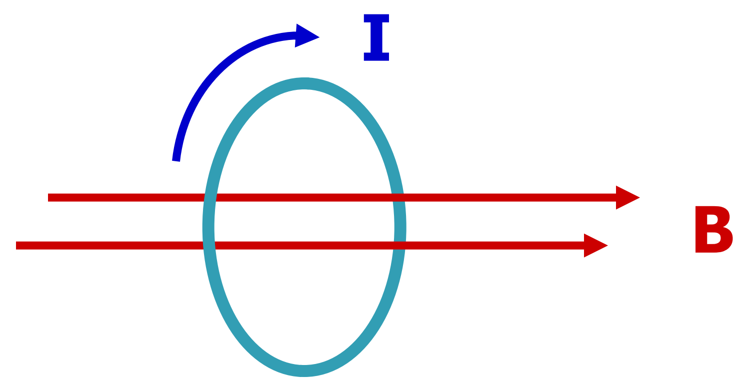
We have found that an electric current can give rise to a magnetic field…

I wonder if a magnetic field can somehow give rise to an electric current…

It is observed experimentally that changes in magnetic flux induce an emf in a conductor.



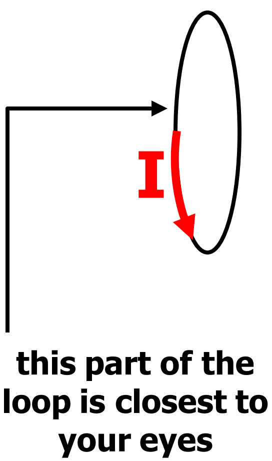
An electric current is induced if there is a closed circuit (e.g., loop of wire) in the changing magnetic flux.

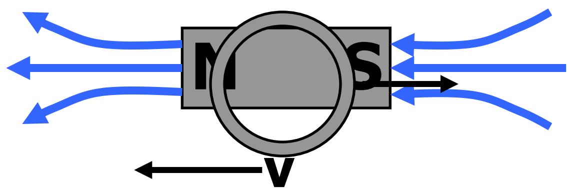


A constant magnetic flux does not induce an emf—it takes a changing magnetic flux.

Note that "change" may or may not not require observable (to you) motion.

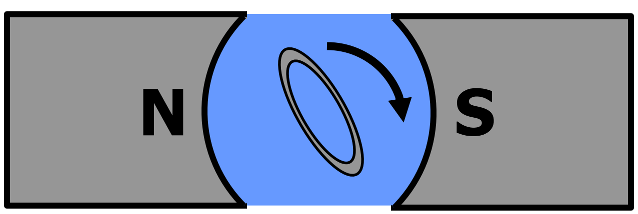
- A magnet may move through a loop of wire, or a loop of wire may be moved through a magnetic field (as suggested in the previous slide). These involve observable motion.



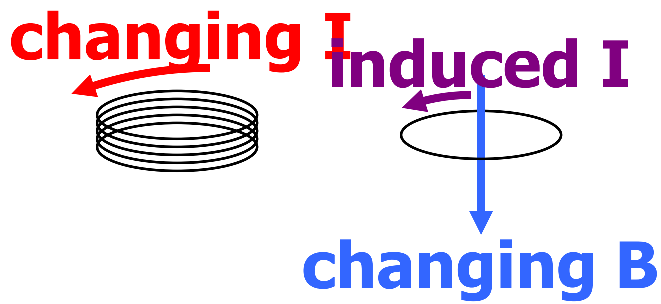


move magnet toward coil

change area of loop inside magnetic field



rotate coil in magnetic field



• A changing current in a loop of wire gives rise to a changing magnetic field (predicted by Ampere's law) which can induce a current in another nearby loop of wire.

• A changing current in a loop of wire gives
by Ampere's law)

In the this case, nothing observable (to your eye) is moving, although, of course microscopically, electrons are in motion.

Induced emf is produced by a changing magnetic flux.

We can quantify the induced emf described qualitatively in the last few slides by using magnetic flux.

Experimentally, if the flux through N loops of wire changes by $d\Phi_B$ in a time dt, the induced emf is

$$\varepsilon = -N\frac{d\Phi_B}{dt}.$$

Faraday's Law of Magnetic Induction

Faraday's law of induction is one of the fundamental laws of electricity and magnetism.

I wonder why the – sign…

In the equation $\mathcal{E} = -N\frac{d\Phi_B}{dt}$,

Faraday's Law of Magnetic Induction

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$ is the magnetic flux.

This is sometimes shown as another version of Faraday's Law:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

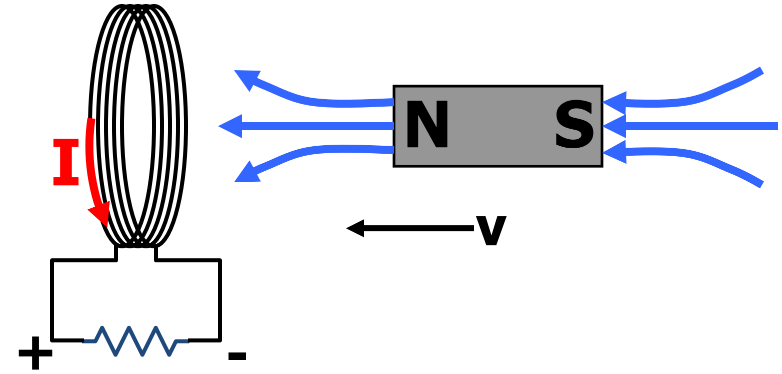
We'll use this version in a later lecture.

In a future lecture, we'll work with $\oint \vec{E} \cdot d\vec{s}$.

Example: move a magnet towards a coil of wire.

N=5 turns   A=0.002 m²

$$\frac{dB}{dt} = 0.4\ T/s$$



$$\varepsilon = -N\frac{d\Phi_B}{dt} = -N\frac{d\int \vec{B}\cdot d\vec{A}}{dt}$$

$$\varepsilon = -N\frac{d(BA)}{dt}$$

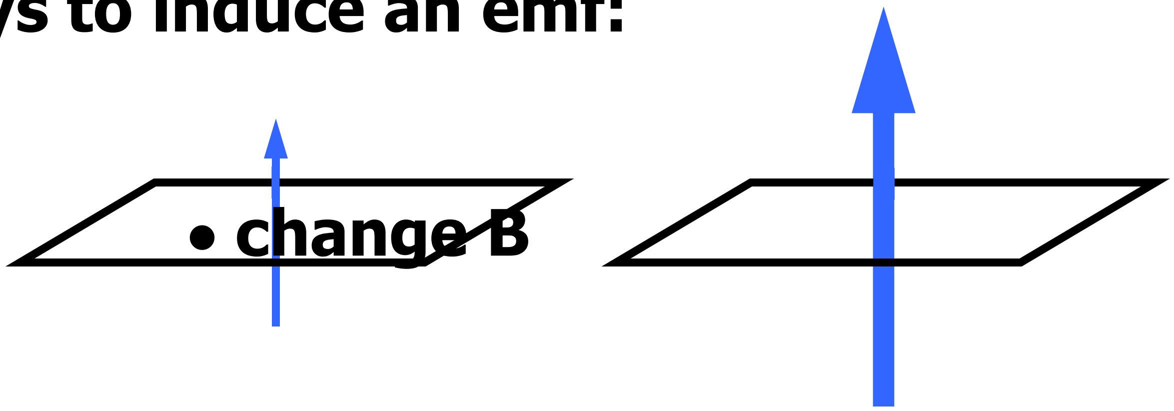
(what assumption did I make here?)

$$\varepsilon = -NA\frac{dB}{dt}$$

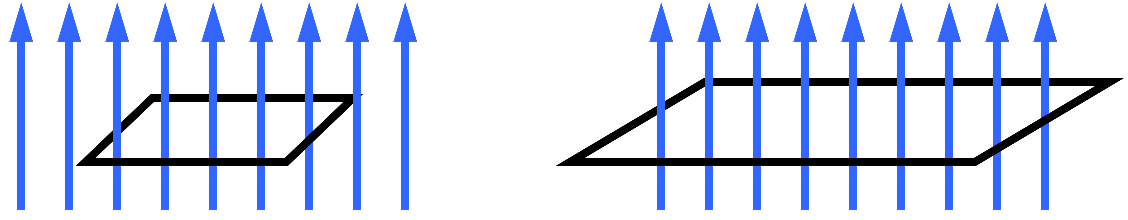
$$\varepsilon = -5\left(0.002\,m^2\right)\left(0.4\frac{T}{s}\right) = -0.004\ V$$

Ways to induce an emf:

- change B

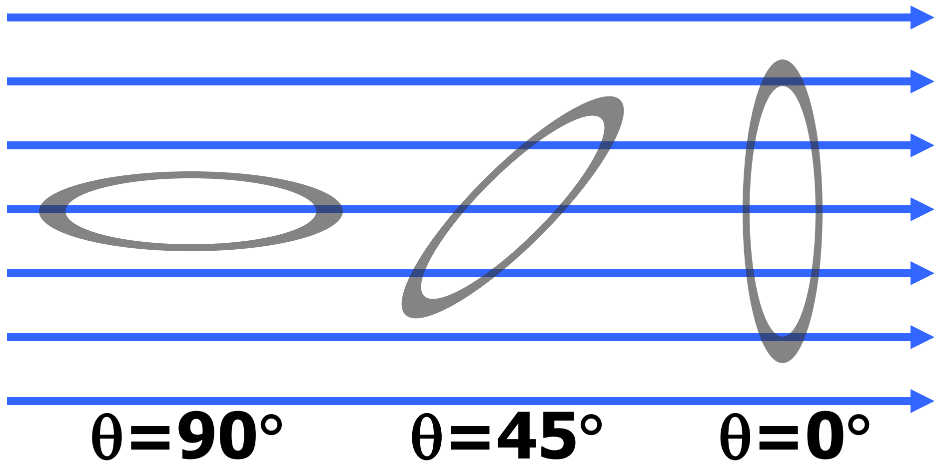


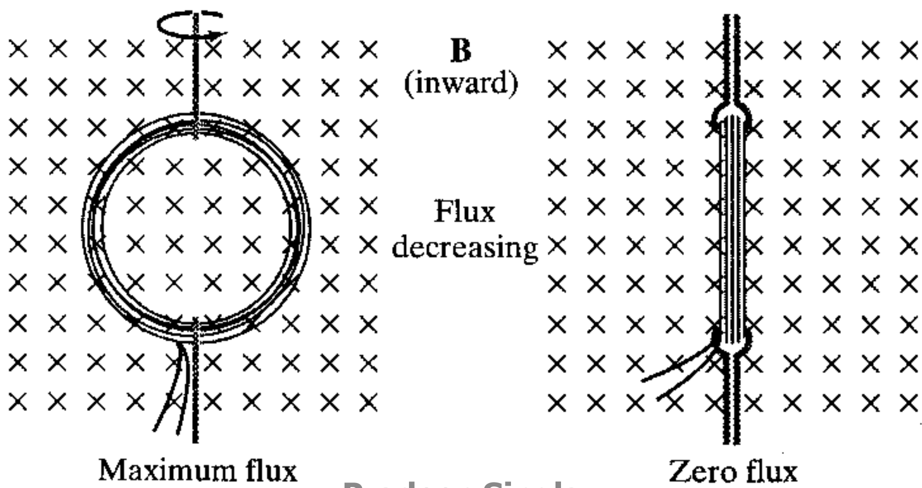
- change the area of the loop in the field



Ways to induce an emf (continued):

- **change the orientation of the loop in the field**





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Example: a uniform (but time-varying) magnetic field passes through a circular coil whose normal is parallel to the magnetic field. The coil's area is 10^{-2} m^2 and it has a resistance of 1 mΩ. B varies with time as shown in the graph. Plot the current in the coil

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -A\frac{dB}{dt}$$

$$\varepsilon = IR \Rightarrow I = \frac{\varepsilon}{R} = -\frac{A}{R}\frac{dB}{dt}$$

For 0 < t < 3:

$$\frac{dB}{dt} = \frac{\Delta B}{\Delta t} = \frac{.01}{3} \Rightarrow I = -\frac{A}{R}\frac{dB}{dt} = -\frac{(.01)}{(.001)}\frac{.01}{3} = -.0333\text{ A}$$

For 3 < t < 5:

$$\frac{dB}{dt} = 0 \Rightarrow I = 0$$

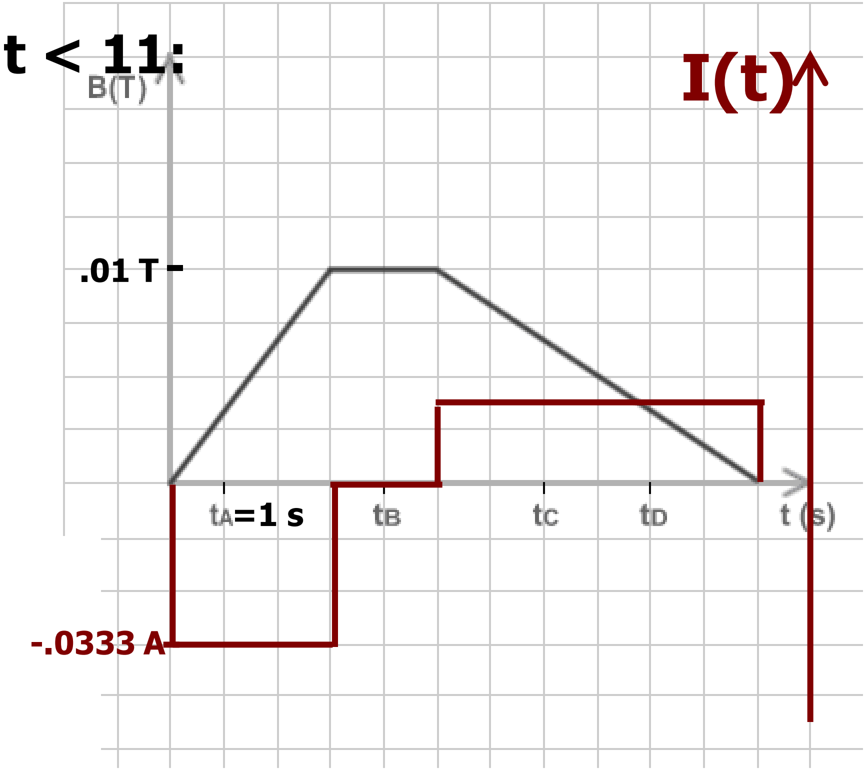
Example: a uniform (but time-varying) magnetic field passes through a circular coil whose normal is parallel to the magnetic field. The coil's area is 10^{-2} m² and it has a resistance of 1 mΩ. B varies with time as shown in the graph. Plot the current in the coil

For 5 < t < 11:

$$\frac{dB}{dt} = \frac{\Delta B}{\Delta t} = \frac{-.01}{6}$$

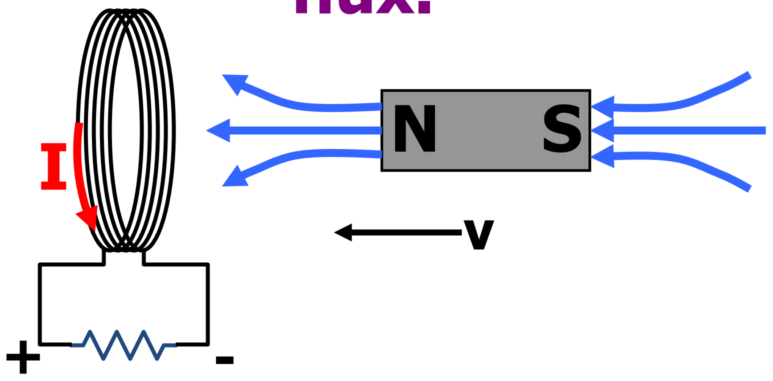
$$\Rightarrow \; I = -\frac{A}{R}\frac{dB}{dt} = -\frac{(.01)}{(.001)}\frac{-.01}{6}$$

$$= +.0167 \text{ A}$$



Experimentally…

Lenz's law—An induced emf always gives rise to a current whose magnetic field opposes the change in flux.*

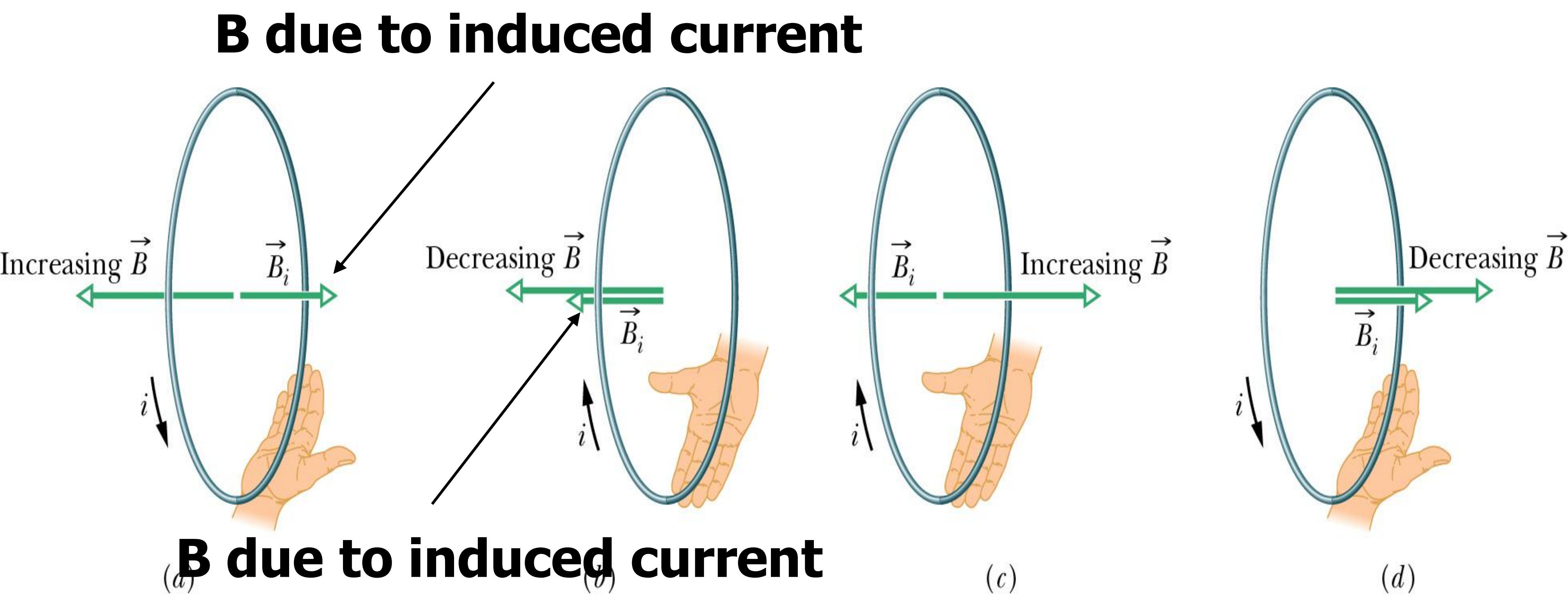


If Lenz's law were not true—if there were a + sign in Faraday's law—then a changing magnetic field would produce a current, which would further increase the magnetic field, further increasing the current, making the magnetic field still bigger…

More on Lenz's Law:

An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current

Question: What is the direction of the current induced in the ring given *B* increasing or decreasing?



Faraday's Law $\varepsilon = -N\frac{d\Phi_B}{dt}$

You can use Faraday's Law to calculate the magnitude of the emf (or whatever the problem wants). Then use Lenz's Law to figure out the direction of the induced current (or the direction of whatever the problem wants).

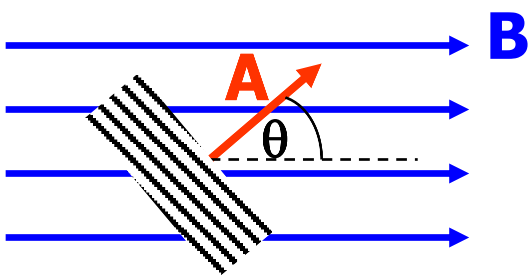
The direction of the induced emf is in the direction of the current that flows in response to the flux change. We usually ask you to calculate the magnitude of the induced emf ($|\varepsilon|$) and separately specify its direction.

Motional emf: an overview

An emf is induced in a conductor moving in a magnetic field.

Your text introduces four ways of producing motional emf. We will cover the first two in this lecture.

1. Flux change through a conducting loop produces an emf: rotating loop.



side view

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

start with this

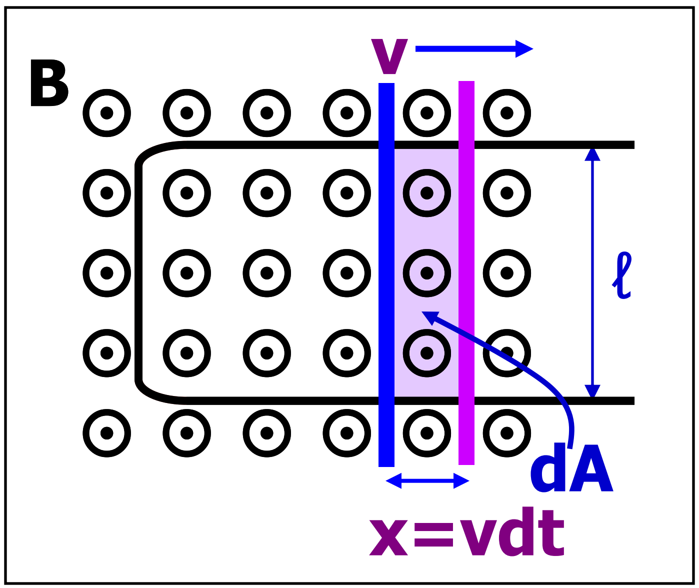
$$\varepsilon = NBA\,\omega \sin(\omega t)$$

$$I = \frac{NBA\omega}{R} \sin(\omega t)$$

$$P = INBA\omega \sin(\omega t)$$

derive these

2. Flux change through a conducting loop produces an emf: expanding loop.



$$\varepsilon = -\frac{d\Phi_B}{dt}$$

$$\vec{F}_M = I\vec{\ell} \times \vec{B}$$

start with these

$$|\varepsilon| = B\,\ell\,v$$

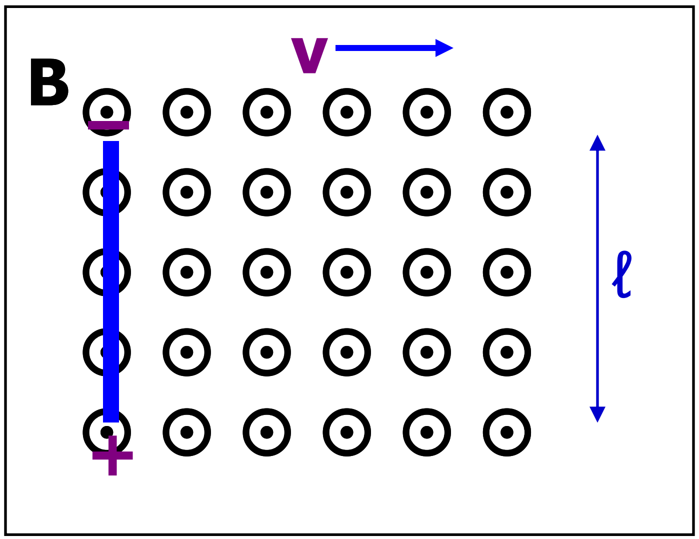
$$I = \frac{\varepsilon}{R} = \frac{B\,\ell\,v}{R}$$

$$P = \vec{F}_P \cdot \vec{v} = I\ell B v$$

derive these

Next time we will look at two more examples of motional emf…

3. Conductor moving in a magnetic field experiences an emf: magnetic force on charged particles.



start with these

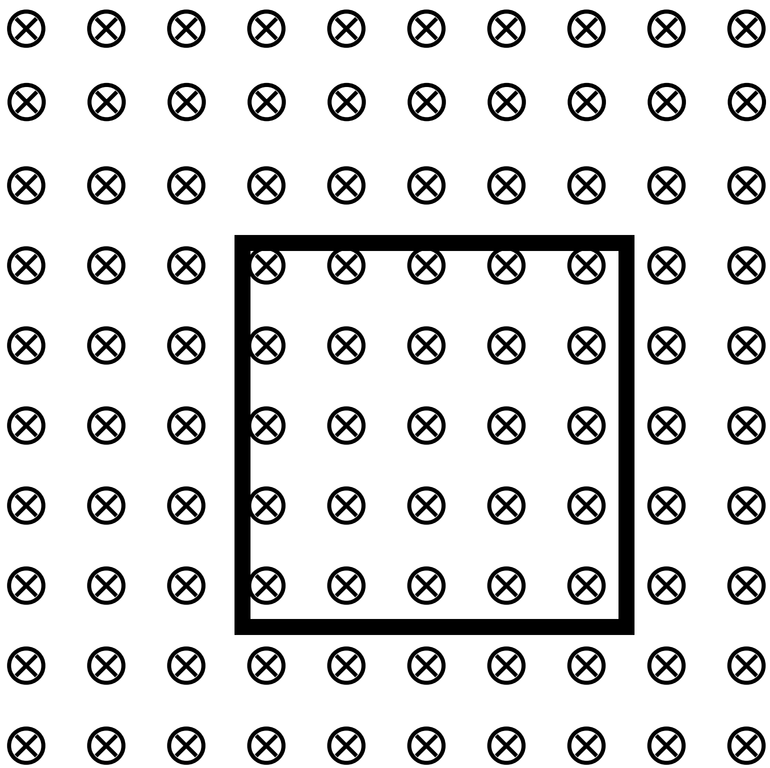
$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$\varepsilon = E\ell \qquad (\text{Mr. Ed})$$

derive this

$$\varepsilon = B\ell v$$

4. Flux change through a conducting loop produces an emf: moving loop.



start with this

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

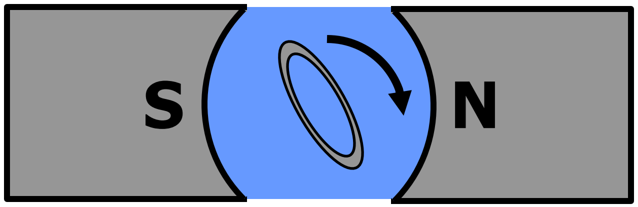
derive these

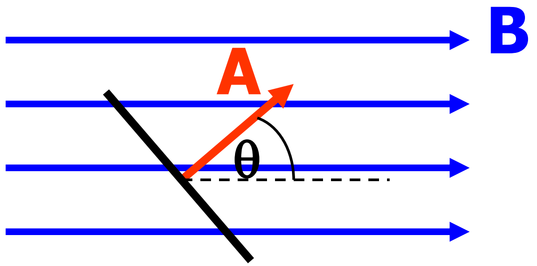
$$\varepsilon = B\ell v$$

$$I = \frac{B\ell v}{R} \qquad P = I\ell B v$$

Generators and Motors: a basic introduction

Take a loop of wire in a magnetic field and rotate it with an angular speed ω.





side view

$$\Phi_B = \vec{B}\cdot\vec{A} = BA\cos(\theta)$$

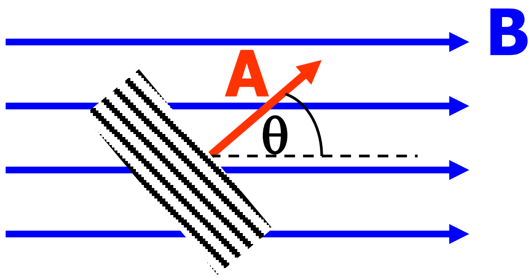
Choose θ_0=0. Then

$$\theta = \theta_0 + \omega t = \omega t\,.$$

$$\Phi_B = BA\cos(\omega t)$$

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Generators are an application of motional emf.



side view

If there are N loops in the coil

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

$$\varepsilon = -N\frac{d\left(BA\cos\left(\omega t\right)\right)}{dt}$$

$$\varepsilon = NBA\,\omega\sin\left(\omega t\right)$$

The NBA equation!

$|\varepsilon|$ is maximum when $\theta = \omega t = 90°$ or $270°$; i.e., when Φ_B is zero. The *rate* at which the magnetic flux is changing is then maximum. On the other hand, ε is zero when the magnetic flux is maximum.

emf, current and power from a generator

$$\varepsilon = NBA\,\omega \sin(\omega t)$$

$$I = \frac{\varepsilon}{R} = \frac{NBA\omega}{R}\sin(\omega t)$$

$$P = \varepsilon I = INBA\omega \sin(\omega t)$$

Example: the armature of a 60 Hz ac generator rotates in a 0.15 T magnetic field. If the area of the coil is $2\text{x}10^{-2}$ m^2, how many loops must the coil contain if the peak output is to be ε_{max} = 170 V?

$$\varepsilon = N B A \omega \sin(\omega t)$$

$$\varepsilon_{max} = N B A \omega$$

$$N = \frac{\varepsilon_{max}}{B A \omega}$$

$$N = \frac{(170\ \text{V})}{(0.15\ \text{T})\left(2\times10^{-2}\ \text{m}^2\right)\left(2\pi\times60\ \text{s}^{-1}\right)}$$

$$\boxed{N = 150 \quad (\text{turns})}$$

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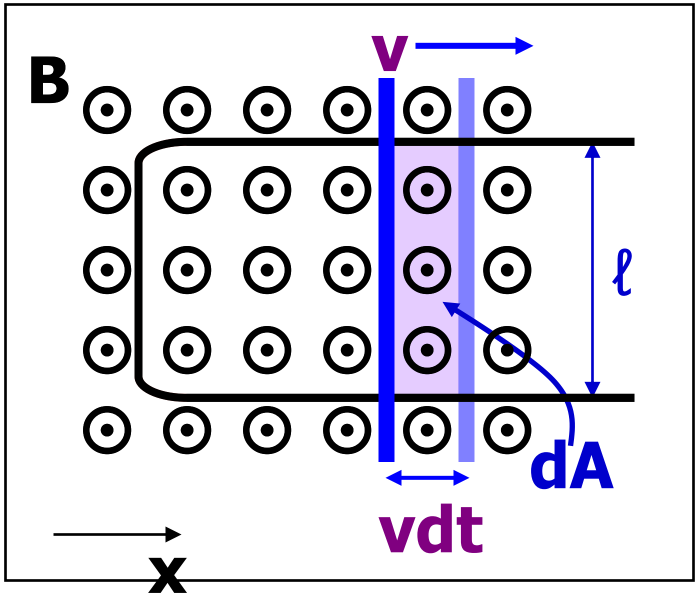
Another Kind of Generator: A Slidewire Generator

Recall that one of the ways to induce an emf is to change the area of the loop in the magnetic field. Let's see how this works.

A U-shaped conductor and a moveable conducting rod are placed in a magnetic field, as shown.
The rod moves to the right with a constant speed v for a time dt.



The rod moves a distance vdt and the area of the loop inside the magnetic field increases by an amount

$$dA = \ell \, v \, dt \, .$$

The loop is perpendicular to the magnetic field, so the magnetic flux through the loop is $\Phi_B = \oint \vec{B} \cdot d\vec{A} = BA$. The emf induced in the conductor can be calculated using Faraday's law:

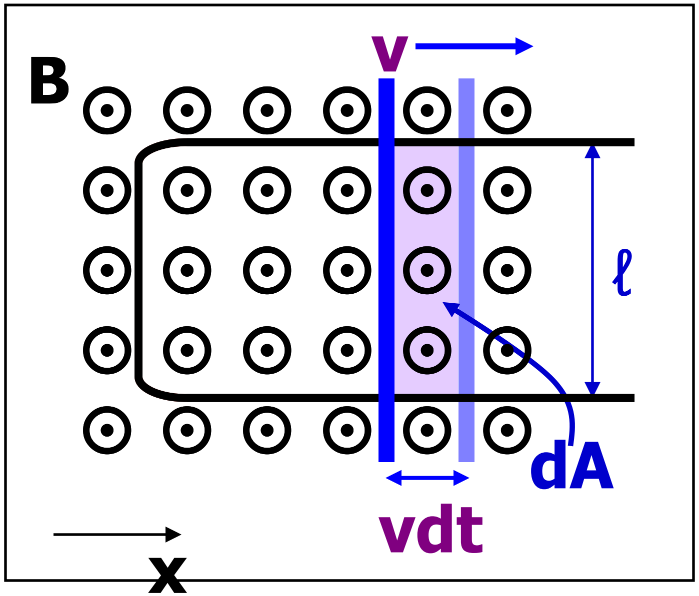
$$|\varepsilon| = \left| -N \frac{d\Phi_B}{dt} \right|$$

$$|\varepsilon| = \left| 1 \frac{d(BA)}{dt} \right|$$

$$|\varepsilon| = \left| \frac{B\, dA}{dt} \right|$$

$$|\varepsilon| = \left| B\ell \frac{dx}{dt} \right|$$

$$|\varepsilon| = B \ell v .$$



B and v are vector magnitudes, so they are always +. Wire length is always +.

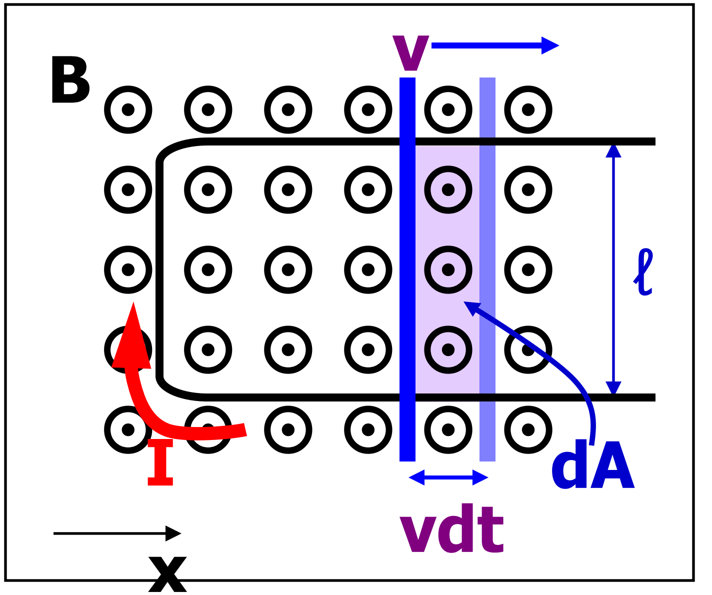
Direction of current?

The induced emf causes current to flow in the loop.

Magnetic flux inside the loop increases (more area).

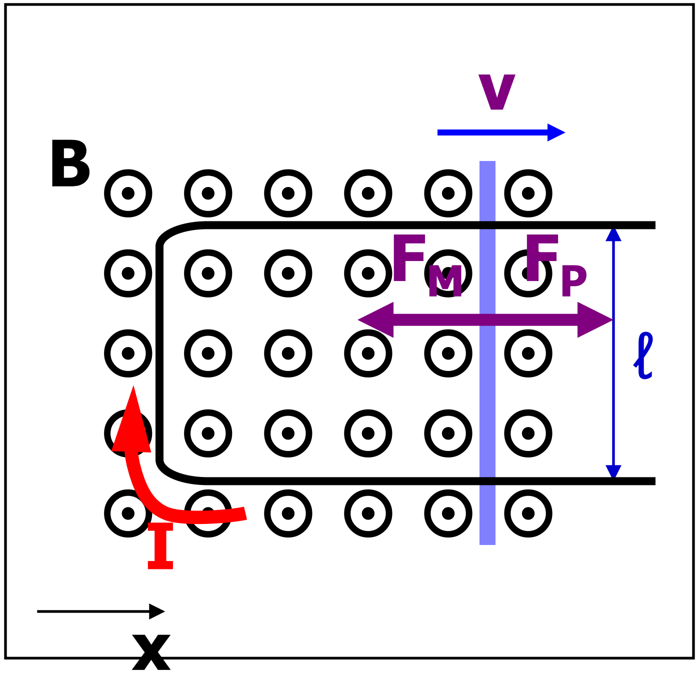
System "wants" to make the flux stay the same, so the current gives rise to a field inside the loop into the plane of the paper (to counteract the "extra" flux).

Clockwise current!



As the **bar** moves through the magnetic field, it "feels" a force

$$\vec{F}_M = I\vec{\ell}\times\vec{B}$$



A constant pulling force, equal in magnitude and opposite in direction, must be applied to keep the bar moving with a constant velocity.

$$\left|\vec{F}_P\right| = \left|\vec{F}_M\right| = I\ell B$$

Power and current.

If the loop has resistance R, the current is

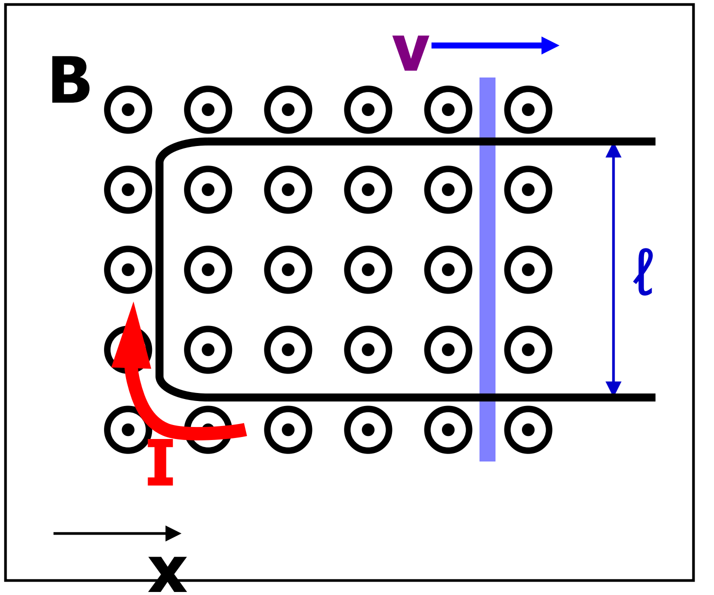
$$I = \frac{\varepsilon}{R} = \frac{B \ell v}{R}.$$

And the power is

$$P = \vec{F}_P \cdot \vec{v} = I\ell Bv$$

$$P = I(IR) = I^2 R$$

(as expected).



Mechanical energy (from the pulling force) has been converted into electrical energy, and the electrical energy is then dissipated by the resistance of the wire.